Is it possible to detect effects of wave optics in gravitational lensing?

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Motivation

- **Context**

  - Wave optics produces a frequency dependent magnification: no magnification for high wavelengths and oscillations for smaller ones.

  - Limiting factor to constrain low-mass PBH abundance from microlensing light-curves (e.g. Niikura et al, 2018)

  - Femtolensing: PBH would generate wiggles in GRB spectra (Gould, 1992). Strong constraints on low-mass PBH (Barnacka et al, 2012), finite source size kills the effect, so that limit does not apply (Katz et al, 2018)

- **Objectives**

  - Establish detectability conditions to have a clear pattern imprinted by the effects of waves in the spectra of astrophysical objects.

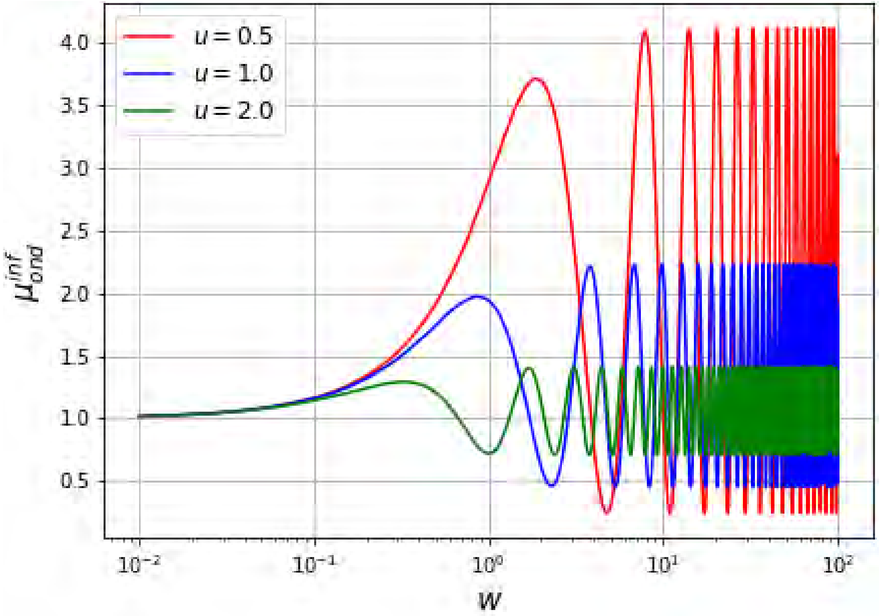
  - We explored ranges of lens masses, source types, and relative distances where we *could detect these effects in the spectrum of different sources* across different scales of the electromagnetic spectrum.

Wave Effects

➢ To account for wave optics, we need to solve the Maxwell's equations in curved spacetime with a perturbation in the potential. For point-like lens and source, the magnification is given by (Schneider et al, 1999):

$$\mu_{ond}^{inf}(w,u) = \frac{\pi w}{1-e^{\pi w}}\left|\,{}_1F_1\left(\frac{iw}{2},1;\frac{iwu^2}{2}\right)\right|^2 \quad \text{(Wave Optics)} \qquad (1)$$

${}_1F_1$ It is a confluent hypergeometric function.



Where:

- Dimensionless frequency: $w = 4\pi(1+z_L)\frac{r_{sch}(M)}{\lambda}$ (2)
- u: impact parameter in units of Einstein radius
- $\omega = 2\pi/\lambda$;
- $r_{sch}(M)$: Schwarzschild radius of the lens;
- M: lens mass;
- λ: wavelength;
- z_L: lens redshift.

- $\lambda \gg r_{Sch}\,(M)$ (that is, $w \ll 1) \Longrightarrow \mu \to 1$

- At high-frequency limits ($w \gg 1$) (Matsunaga, N; Yamamoto, K, 2006)

$$\mu_{int}^{inf}(w,u) = \frac{u^2+2}{u\sqrt{u^2+4}} + \frac{2}{u\sqrt{u^2+4}}\sin\{wf(u)\} \quad \text{(Eikonal Limit)} \qquad (3)$$

(Point source term + interference term)

where $f(u) = \left(\frac{1}{2}u\sqrt{u^2+4} + ln\left(\frac{\sqrt{u^2+4}+u}{u-\sqrt{u^2+4}}\right)\right)$.

Wavelensing Effects on the Spectrum of Astrophysical Sources

- Oscillations will Impact the spectrum of sources → wave pattern → **"WAVELENSING"**

- We can explore the influence of lensing on the spectrum for different mass scales of the lens that are sensitive to different wavelengths.

• Effects of lensing on the spectrum (wavelensing): $\boldsymbol{f_I(\lambda) = \mu(\lambda) \times f_S(\lambda)}$

• $\boldsymbol{f_S(\lambda)}$: spectrum of a source.

• $\boldsymbol{f_I(\lambda)}$: processed spectrum (spectrum of the image).

- Define "detectability conditions" so that the wave pattern can be easily identified in the spectra

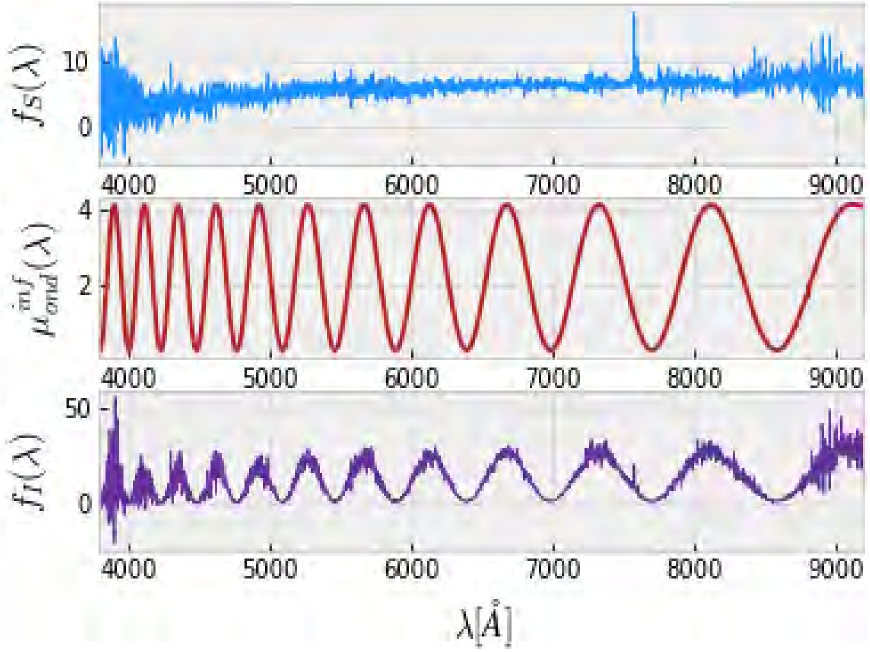
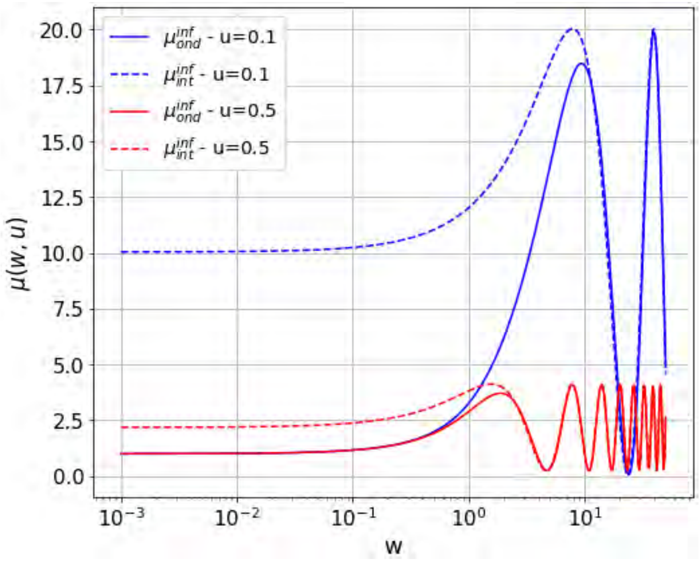


Figure: Spectrum processed due to wavelensing.

Detectability Conditions

1. **Condition on the minimum amplitude:**

$$\frac{\Delta\mu}{\mu} = \frac{(\mu_{peak} - \mu_{valley})}{\mu_{mean}} > 0.1$$



- ➢ This condition establishes that $u < u_\mu$. Where u_μ corresponds to the equality above.

2. **Condition for being in the oscillatory regime:**

$$\lambda_{typical} < \lambda_{peak}$$

- ➢ This condition establishes that $u > u_{peak}(\lambda_{typical}, M)$.

3. **Condition due to resolution:**

- We need the oscillation to occur within a wavelength range larger than the spectrograph resolution $\delta\lambda$. In spectroscopy, it is common to define resolution in terms of $R = \lambda/\delta\lambda$.

- ➢ This condition establishes that $u < u_R(R, \lambda, M)$.

Detectability Conditions

4. <u>Condition for the existence of oscillation in the observed spectrum:</u>

- We need at least one complete oscillation within the wavelength range (λ) of the spectrograph.

➢ This condition establishes that $u > u_{\Delta\lambda}(\Delta\lambda, M)$, where $\Delta\lambda = \lambda_{max} - \lambda_{min}$.

- These conditions were obtained in terms of 'u' as it gives us an idea of the probability of the event occurring.

- We consider some specific instruments in each spectral range as representative of the state-of-the-art in terms of coverage and resolution.

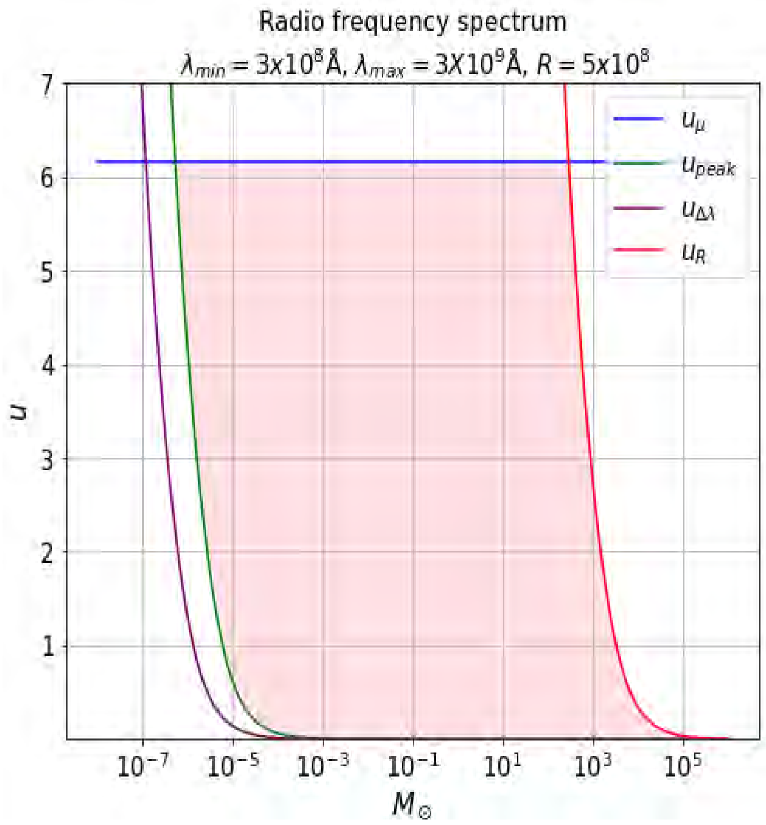
Instrument	Band	λ(Å) range	Spectral resolution
SETI	Radio	$[3 - 30]\times10^8$	5×10^8
ALMA	Submm	$[3 - 35]\times10^6$	3×10^7
VLT (CRIRES)	IR	$[1 - 5]\times10^4$	1×10^5
VLT (FLAMES)	Optical	3700 - 9500	4×10^4
HST (GHRS)	UV	1150 - 3200	1×10^5
Chandra X-Ray (LETG)	X-ray	50 - 160	1×10^3
INTEGRAL (SPI)	γ-ray	10^{-3} - 0.1	5×10^2

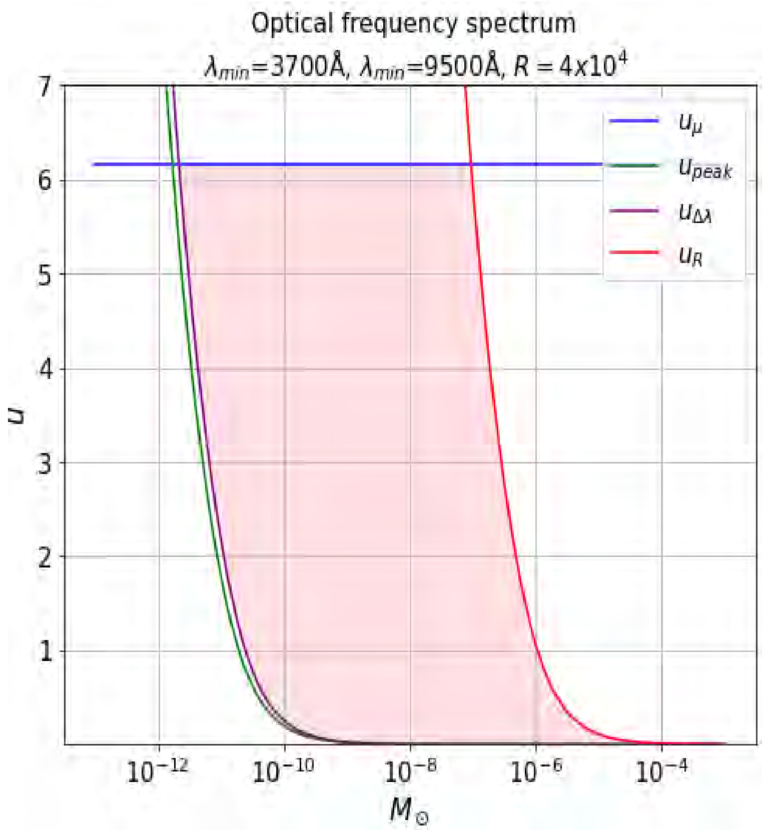
Detectability Conditions

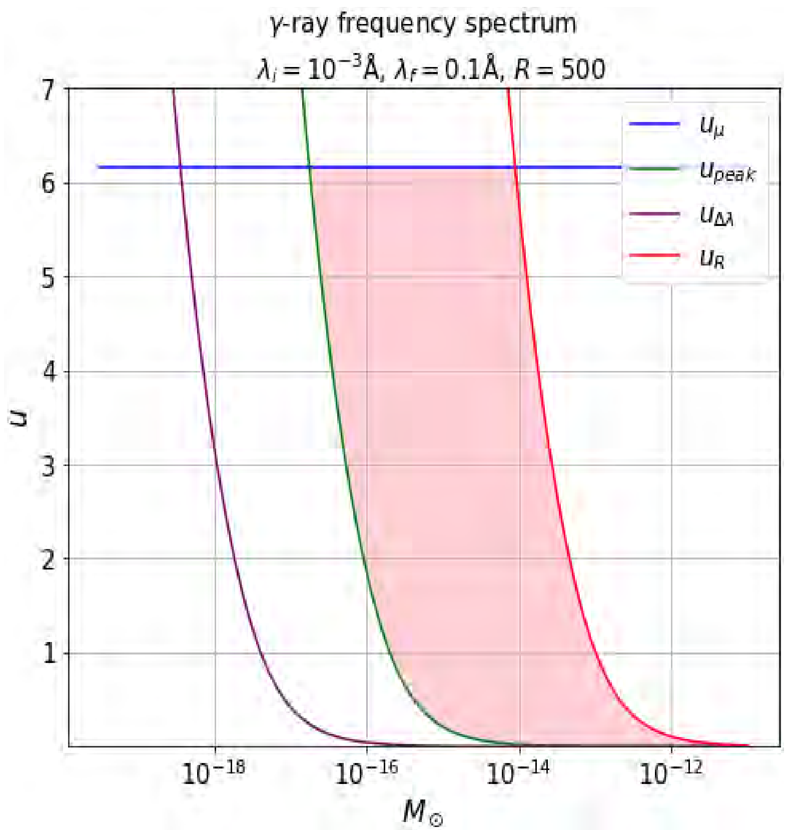
- Allowed regions in the $u \times M$ plane for each band of the electromagnetic spectrum.

  - Solid lines obtained under point source detectability conditions;

  - The pink area indicates the wavelensing detection region.
  - Each spectral range is sensitive to different lens masses.







Finite Source Effect

- To model the finite source effect, we assume that the source is a disk with constant surface brightness. In this case, the magnification is given by (Schneider et al, 1999):

$$\boldsymbol{\mu(u,r,w) = \frac{1}{\pi r^2} \int d^2\vec{y}\, \mu(w, |u - \vec{y}|)} \qquad (4)$$

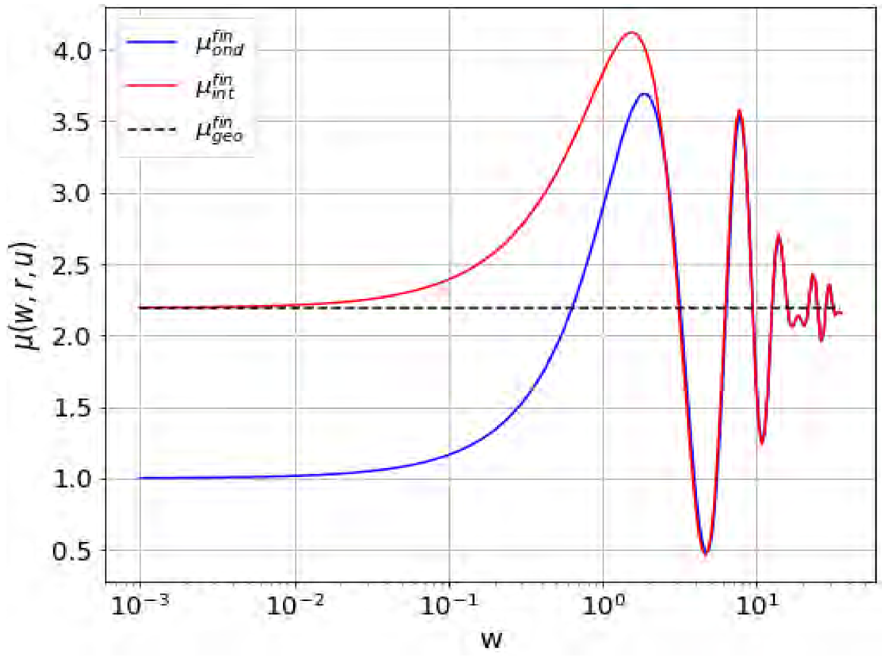
  - $r = \frac{r_S}{D_S \theta_E(M, D_L, D_S)} = r_S \left(\frac{c^2 D_L}{4GMD_{LS}D_S} \right)^2$
  - r_S: radius of the source in physical distance units;
  - D_L: observer-lens angular diameter distance
  - D_S: observer-source angular diameter distance
  - $\vec{y}$ corresponds to a generic point in the source disk.

❑ The finite source effect destroys oscillations at high frequencies and is stronger as the radius becomes larger relative to the impact parameter.

➢ $r < u$: oscillations are preserved in a given frequency range.

➢ $r \geq u$ finite source effect dominates.

$u = 0.5 \;-\; r = 0.01$



- Considering only $r < u$, we are able to obtain a new analytical result in the small-radius approximation.

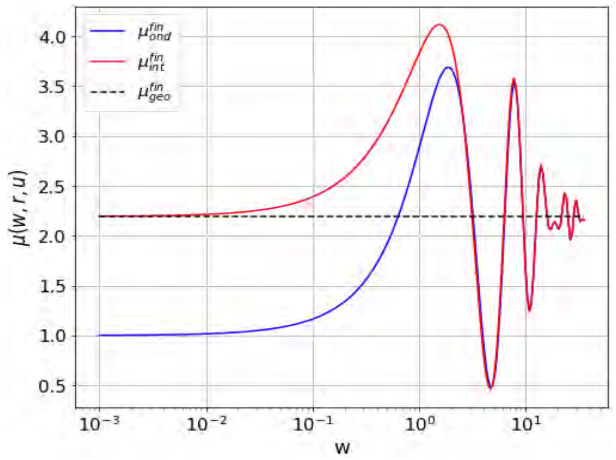
$$\mu^{r^2}_{int}(u,r,w) = \frac{u^2+2}{u\sqrt{u^2+4}} + \frac{2}{u\sqrt{u^2+4}} \sin\{wf(u)\} + r^2 \left(\frac{4(u^2+1)}{u^3(u^2+4)^{5/2}} + \frac{\beta(u,w)}{(u^2+4)^{5/2}} \sin\{wf(u)\} + \frac{\alpha(u,w)}{(u^2+4)^2} \cos\{wf(u)\} \right) \qquad (5)$$

Detectability Conditions (with Finite Source Effect)

1’. <u>Condition on the minimum amplitude</u> (including finite source effect)

- We will use the approximate magnification for small source size to determine the fractional difference, similar to the first condition.

$$\frac{\Delta\mu_{int}^{r^2}}{\mu} = \frac{(\mu_{max} - \mu_{min})}{\mu_{mean}} > 0.1$$



➢ This condition establishes that $u < u_{finite}(\lambda_{min}, M, r_S, D_L, D_S)$.

- We consider typical sizes of sources at distances in two broad distance ranges.

Source	r_S
Neutron Star	$\sim 10^4$m
White Dwarf	$\sim 10^6$m
Main Sequence Star	$\sim R_\odot$
Kilonova	$\sim 10^5$m
sGRB e GRB (short/large γ-ray burst)	$\sim [10^8, 10^9]$m
Supernova	$\sim 10^{12}$m

Local Universe

- Local Universe objects: $z_S \sim 0 \rightarrow D_S = [1, 1000]$kpc;

Cosmological Universe

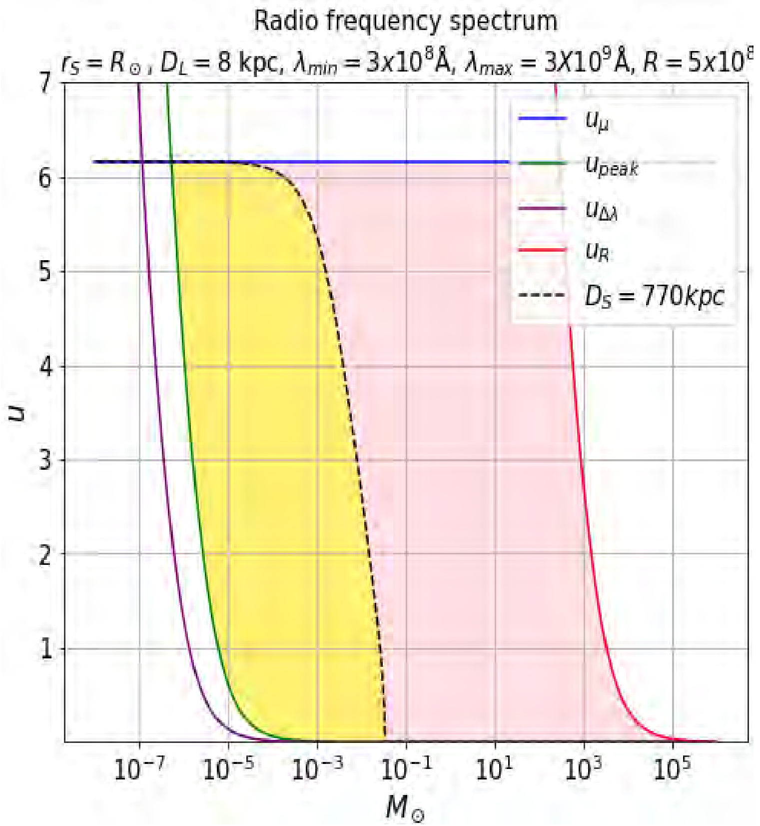
- Cosmological Universe objects: $z_S \sim [0.1, 5] \rightarrow D_S \sim [0.5, 1.5]$Gpc.

z_S: source redshift.

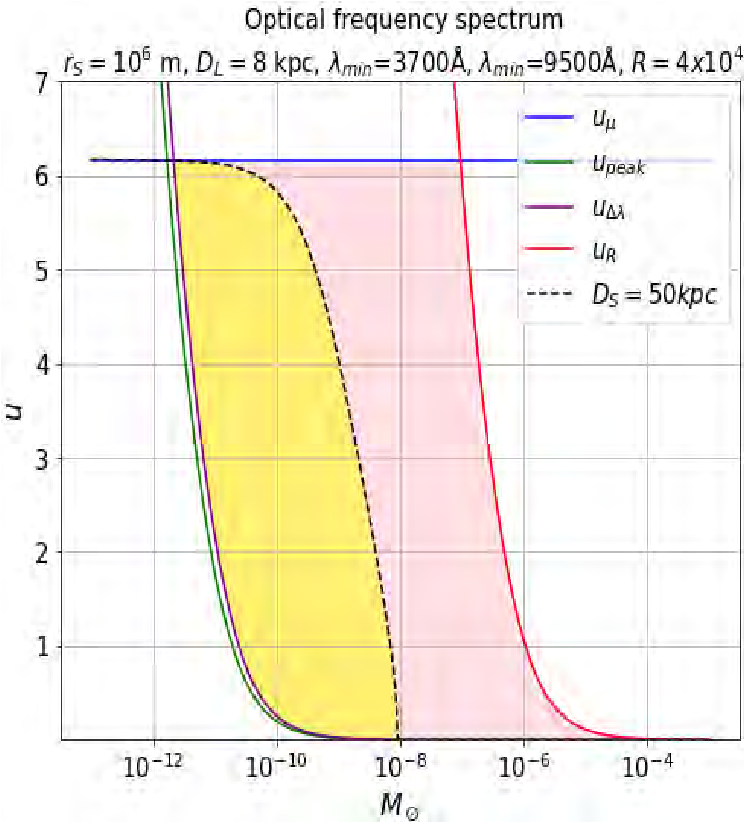
Detectability Conditions (Finite Source Effect)

- Allowed regions in the $u \times M$ plane for each band of the electromagnetic spectrum.

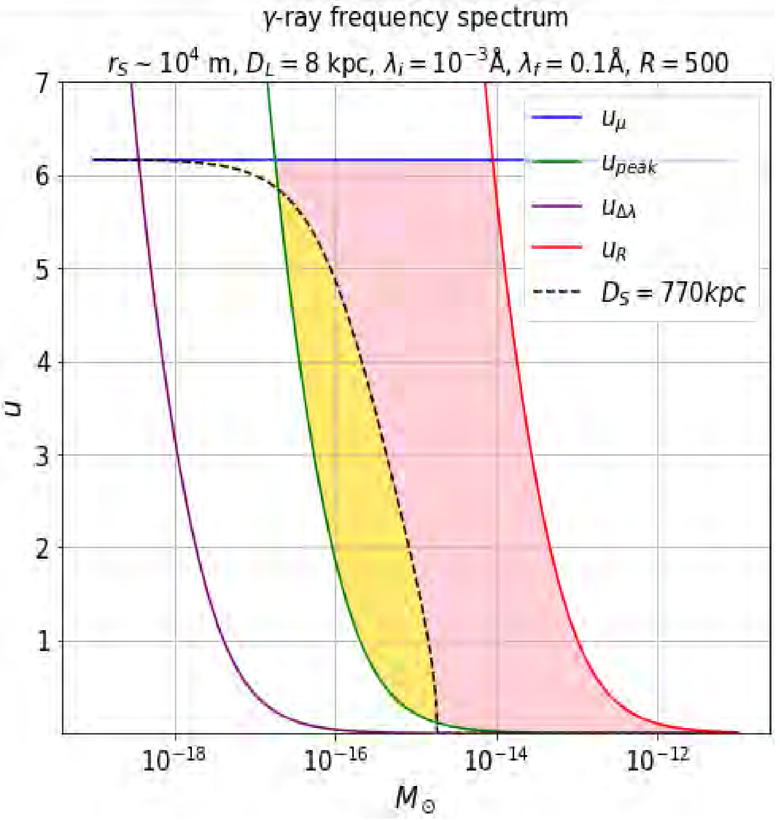
  - Solid lines obtained under point source conditions;
  - Dashed lines obtained under finite source conditions.



- **Source**: Main Sequence Star



- **Source**: White Dwarf



- **Source**: Neutron Star

General Conclusions

- We conclude that it might be possible to detect the wavelensing pattern in the spectrum for astrophysical sources within some ranges of $D_L, D_S, r_S, M, \lambda$ and u.

- For neutron star and white dwarf:
  - Greater chances to detecte wavelensing in almost every spectrum, except γ-ray for white dwarf.
  - Neutron star: from γ-ray to radio with $M \sim [10^{-15} - 10^{3}]M_{\odot}$, when $D_L \sim 8$ kpc.
  - White dwarf: from X-ray to radio with $M \sim [10^{-10} - 10^{-1}]M_{\odot}$.
- Main sequence star:
  - Detectable in the submm and radio, with planetary mass range, $M \sim [10^{-6}, 10^{-3}]M_{\odot}$, when $D_L \sim 8$ kpc.
  - Possibility of using real point-like lenses, such as free floating planets.

- sGRB and GRB:
  - Detection of wavelensing with lenses located in the local universe $D_L \sim 8$ kpc, with $M \sim [10^{-13} - 10^{-10}]M_{\odot}$.
  - Observations in the X-ray spectrum.

- Supernova:
  - There is no possibility of detection at any scale of the spectrum.
- Kilonovae:
  - The beginning of binary merger. → greater detection at all wavelengths as it approaches a point source.
  - From γ-ray to radio with $M \sim [10^{-13} - 10^{3}]M_{\odot}$, when $D_S \sim 1.5$ Gpc

Future Perspectives

- Simulate wavelensed spectra from real unlensed ones an assess the capability of extracting the signal.

- To determine the wavelensing event rate as a function of the characteristics of sources, lenses, and observations:

  - For known lens populations: seek if effect would be detectable;

  - For unknown populations (e.g. PBH): no observation of the effects may place constraints on the lens abundance

- Developing a strategy to observe the phenomenon.

Thank You!

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