


清華大學天文系
Department of Astronomy, Tsinghua University

Implementation of Automatic Differentiation in Microlensing Light Curve Calculation

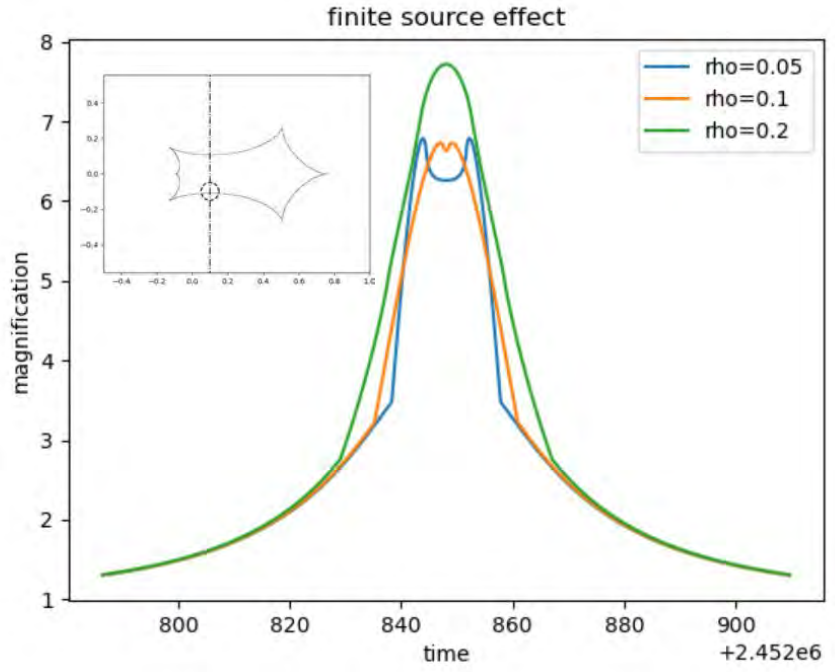
Haibin Ren Supervisor: Wei Zhu

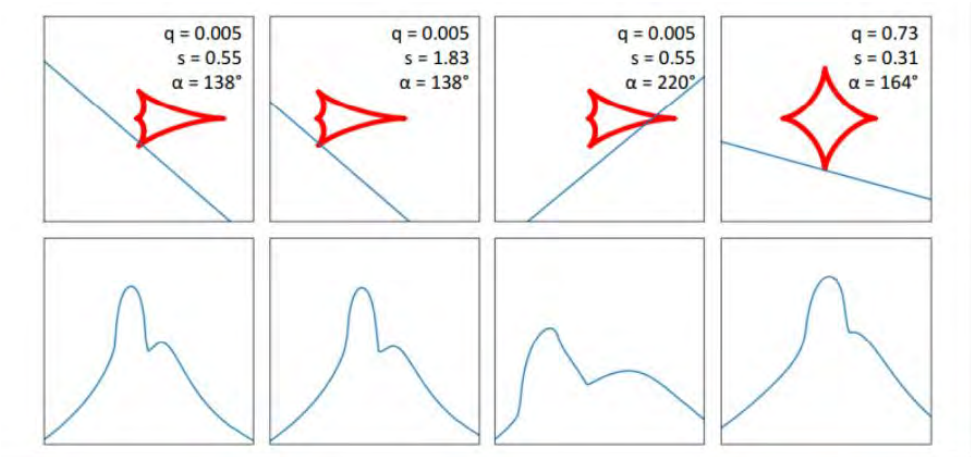
rhb23@mails.tsinghua.edu.cn

Department of Astronomy, Tsinghua University

Background

- Binary lens modelling Challenge:
  - Light curve calculation: roots finding, **finite source effect**
  - Inference: **degeneracy and similarity**
- Current algorithm:
  - VBBinaryLensing contour Integral + optimal sampling
  - Grid search + MCMC inefficient, local minimum
- Advanced algorithm : global optimization? variant of MCMC? Machine Learning?
  - **Optimal sampling +Gradient**

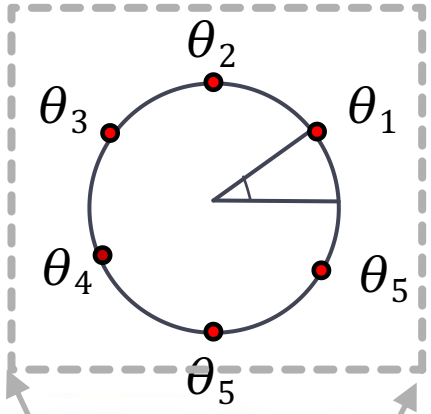




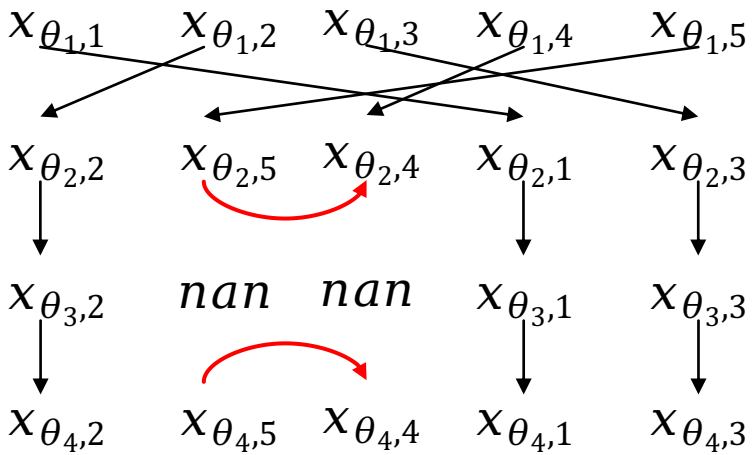
Degeneracy and similarity in binary lens Credit Jennifer Yee

Contour Integral + optimal sampling

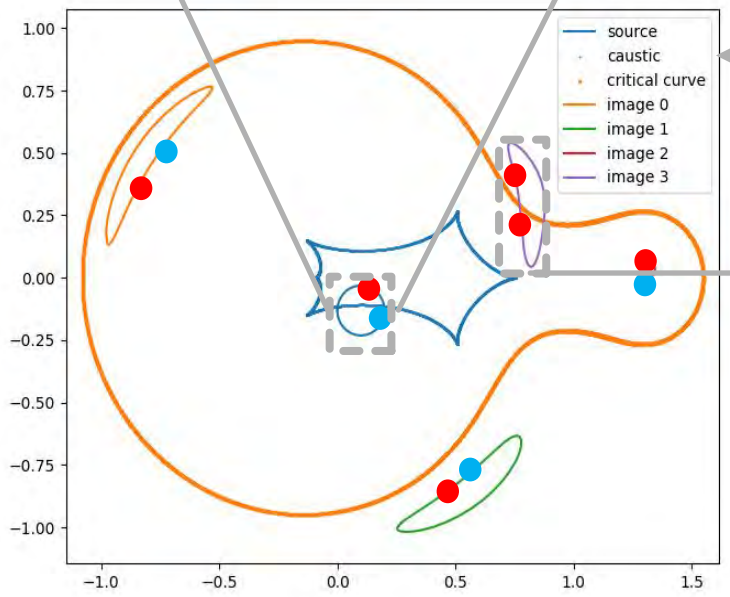
Schramm and Kayser, (1987)
Gould and Gaucherel ,(1997)
Dominik (2007)
Bozza (2010), Bozza et al. (2018)
Bartolić (2023) …



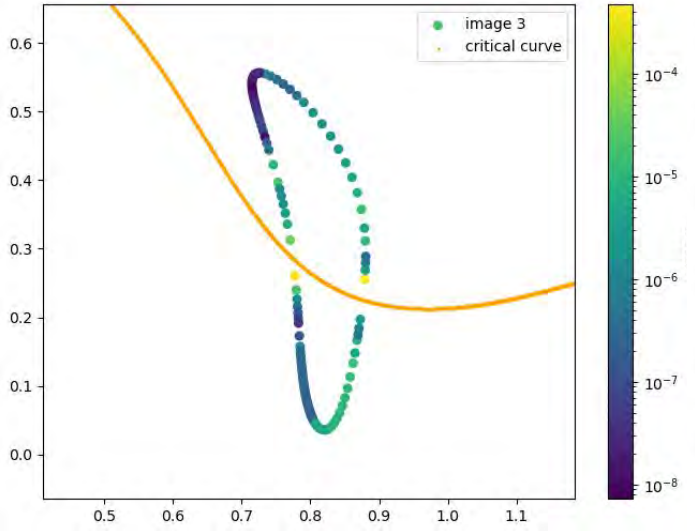
Lens equation

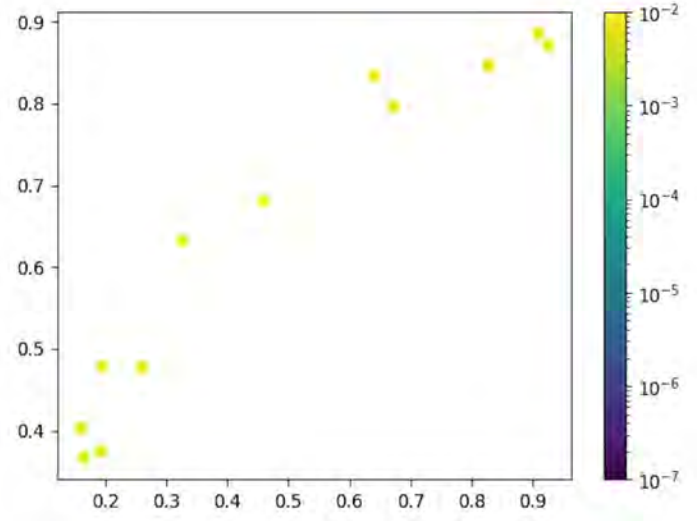


Connect image



Estimate error

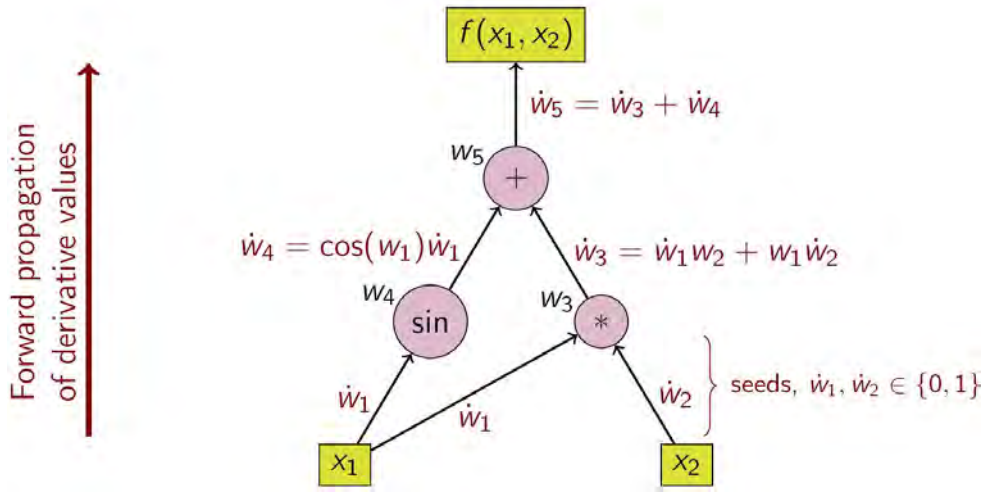




Implementation: Jax

- Automatic differentiation

Decompose the algorithm and use chain rule to get accurate gradient.



- JIT(just-in-time) compile: accelerate
- compile requirement:

Shaped array (we must know the shape of array before code runs): memory and time waste

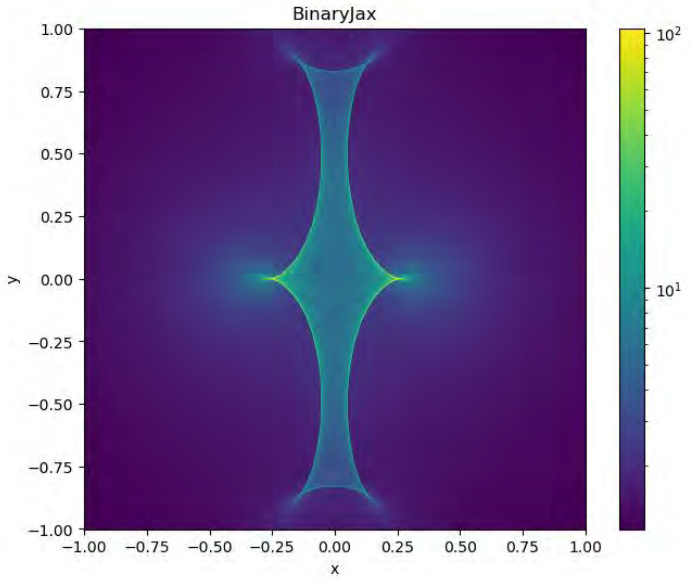
0	θ_i	2π	Nan	Nan	Nan	Nan	Nan	Nan

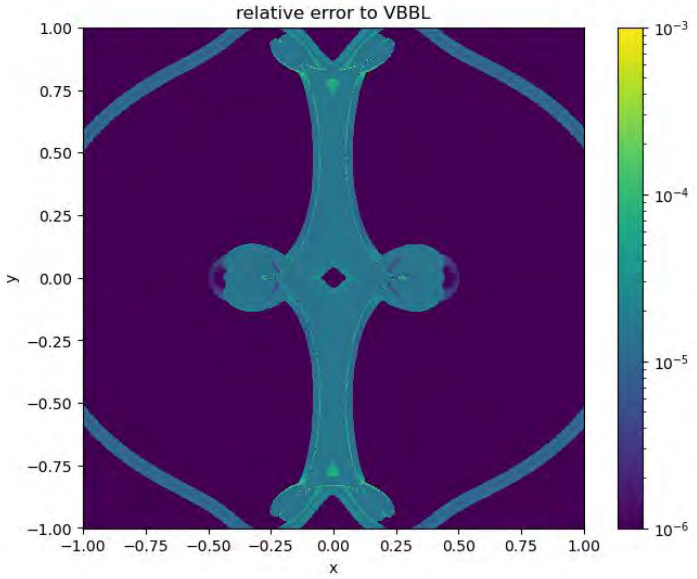
Operation

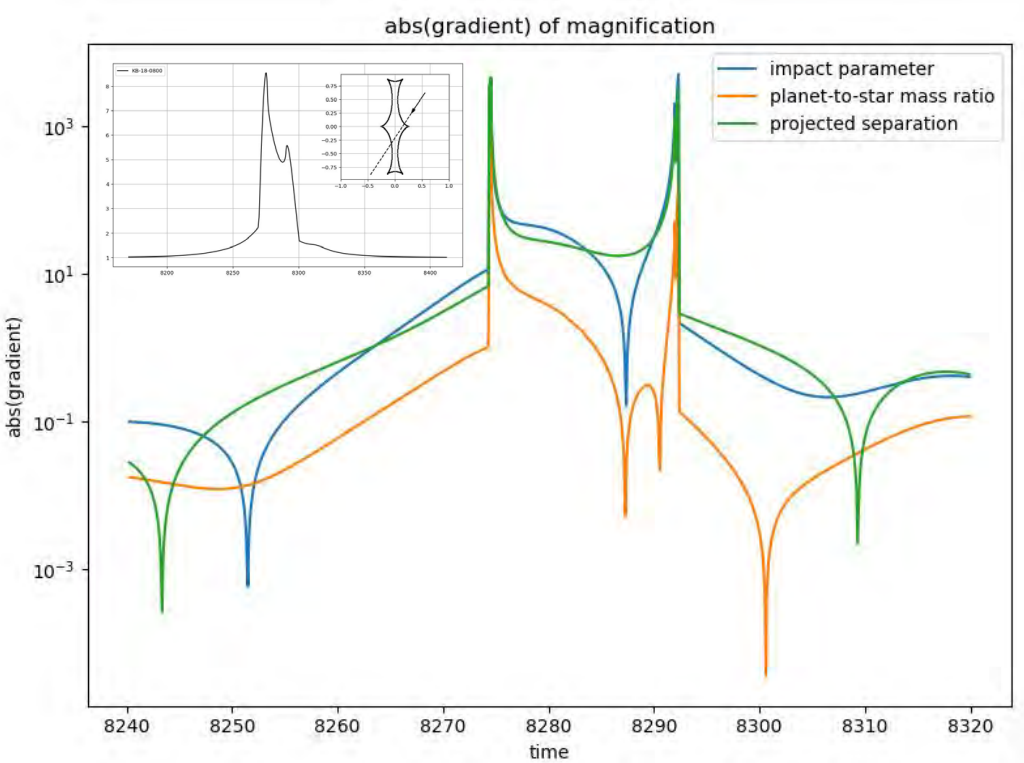
0	θ_i	2π	Nan	Nan	Nan	Nan	Nan	Nan

Result

- Draw magnification maps and compare with VBBL. $q \in [10^{-6}, 1]$, $s \in [0.1,4]$, $\rho \in [10^{-3}, 10^{-1}]$
- **Fast robust and accurate light curve calculation with gradient function**
- ~s for ~light curve with 1e4 points







Application of gradient

- Optimization:

Local minimization: from simplex to gradient based method (CG,BFGS…)

Global minimization: Basin hopping

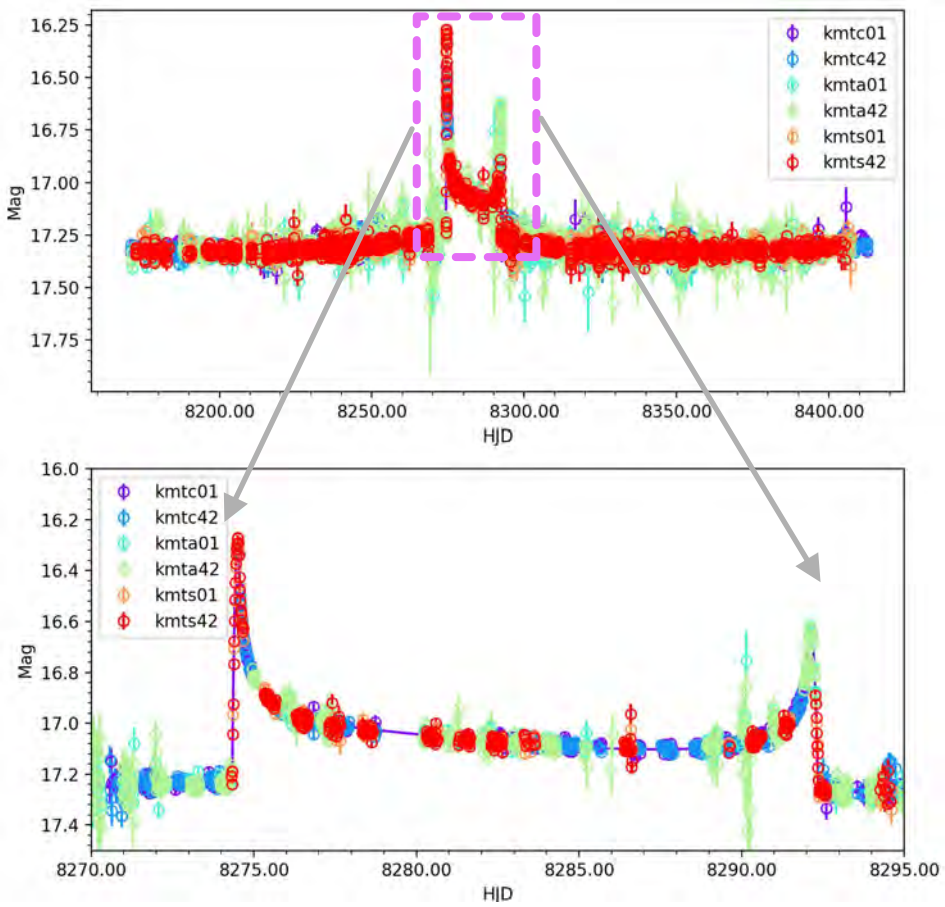
- Posterior:

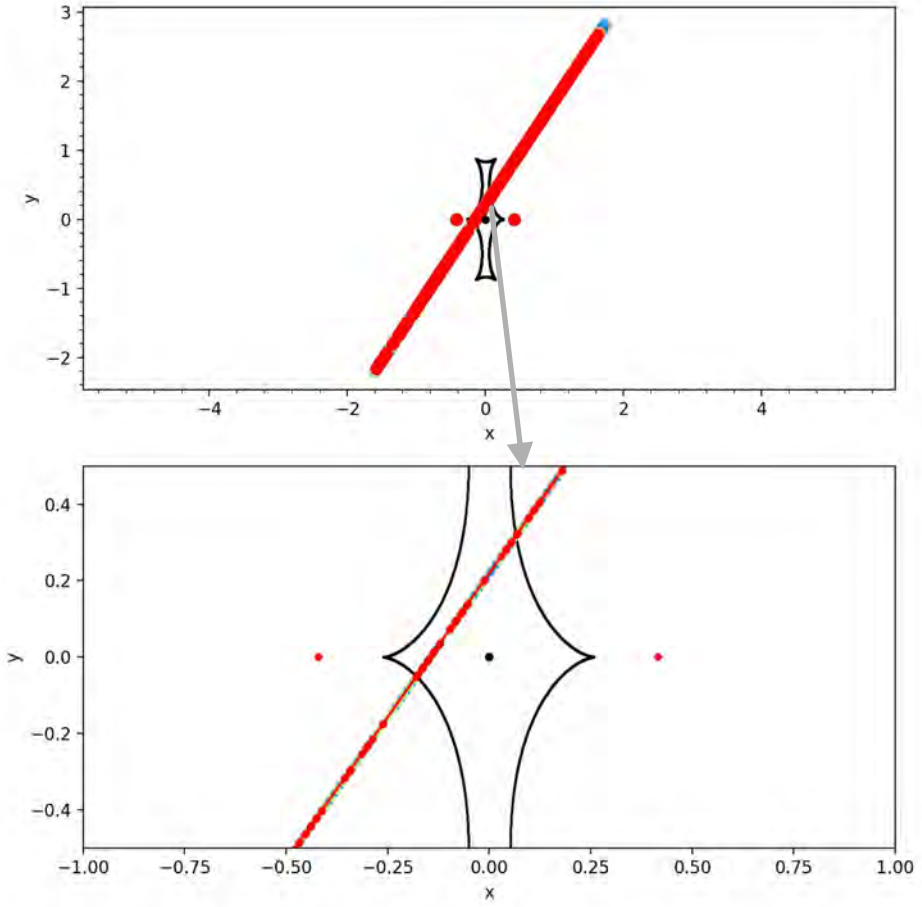
From MCMC to Fisher matrix

From MCMC to variant of MCMC(such as HMC)

Case study: KB-18-0800

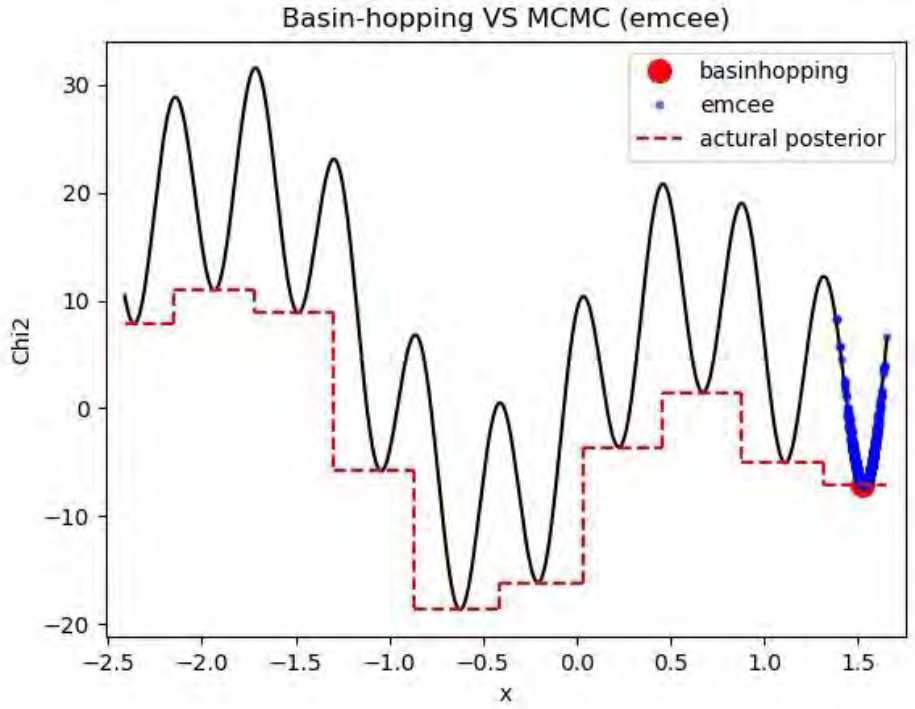
dof :11994 chisq: 12023





Optimization: Basin hopping

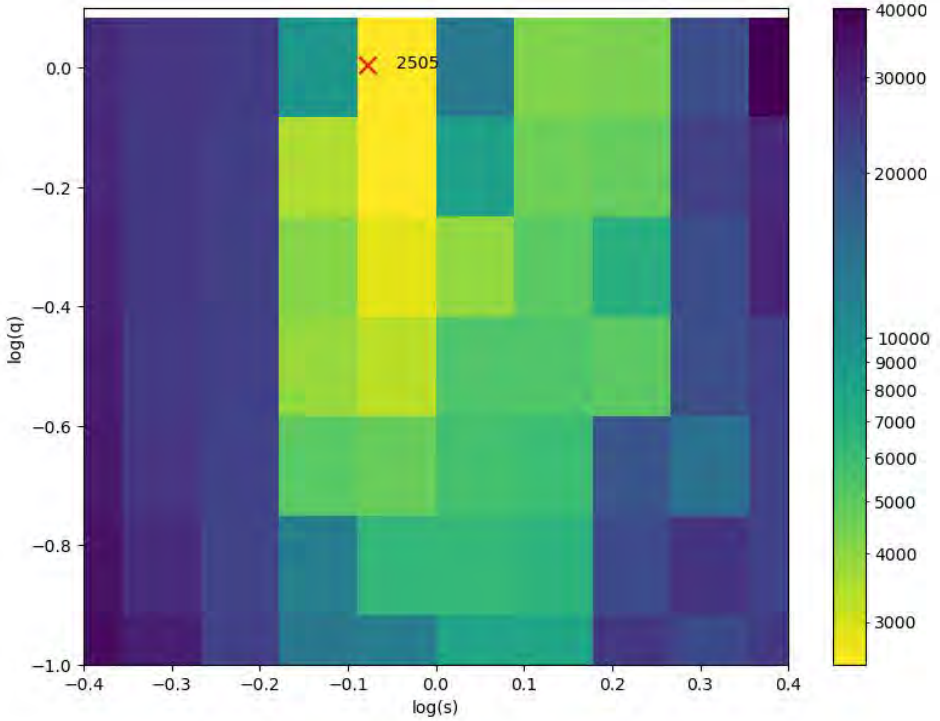
- *RandomPerturbation()*
- *LocalMinimisation() gradient*
- *Accept / Reject (Metropolis criterion)*



- **Grid search**

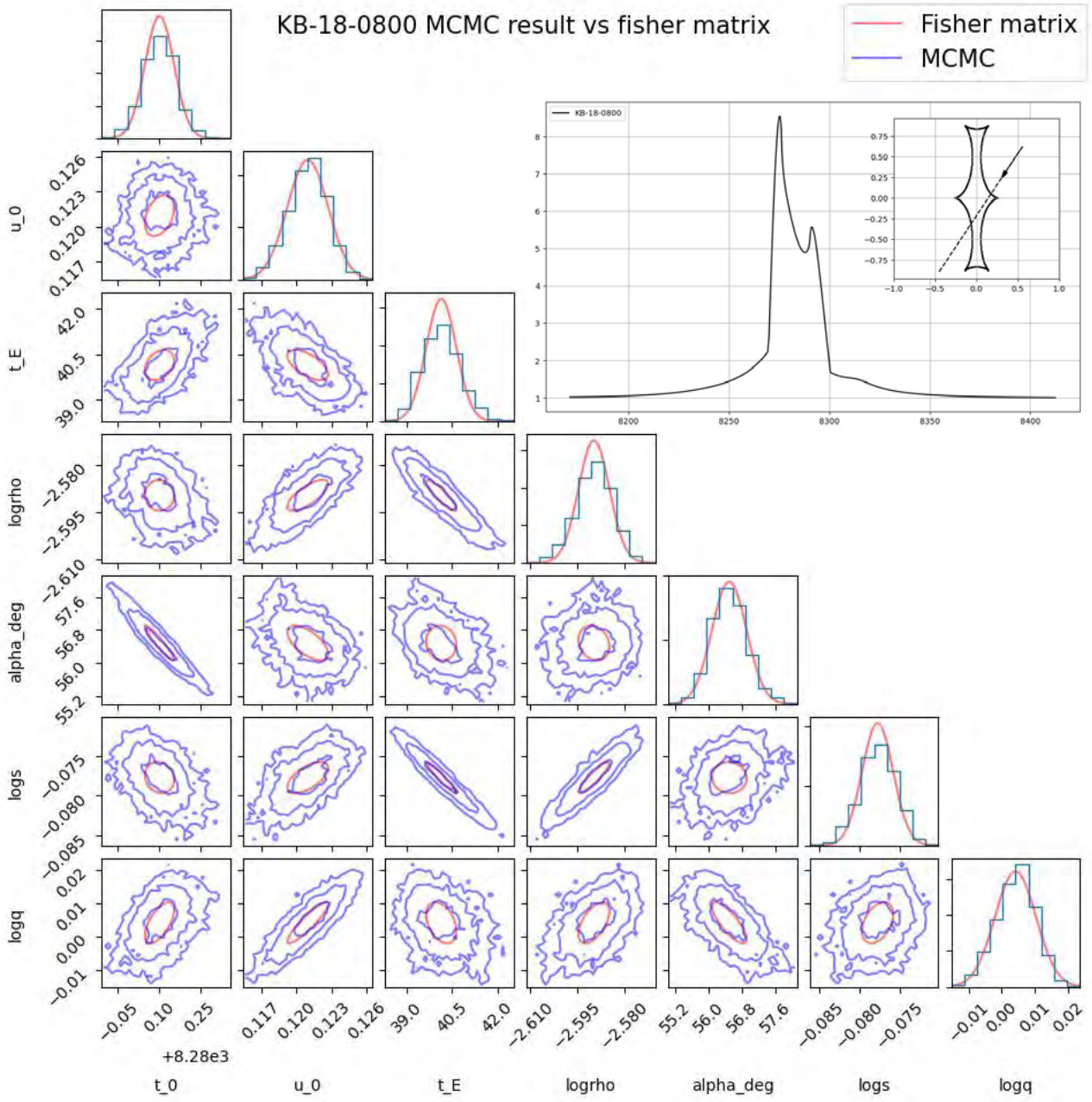
~1000 function calls/grid :basin hopping

~10000 function calls/grid :emcee



Fisher Information Matrix

- $F_{ij} = \Sigma_k \frac{\partial F_{t_k}}{\partial \theta_i} * \frac{\partial F_{t_k}}{\partial \theta_j} * \frac{1}{\sigma_{t_k}^2}$
- Optimization + Fisher information : directly get posterior
- only the linear correlation (Gaussian) Fail when parameters are not well constrained, Sampling: variant of MCMC



Hamiltonian Monte Carlo

- Associate PDF with energy $p(x,q) = \frac{1}{Z} e^{-H(x,q)}$
- Using Hamiltonian dynamic to propose new state. Reversibility and acceptance rate =1.0(theoretically)

weaker autocorrelation, Higher acceptance ratio

MCMC VS. HMC

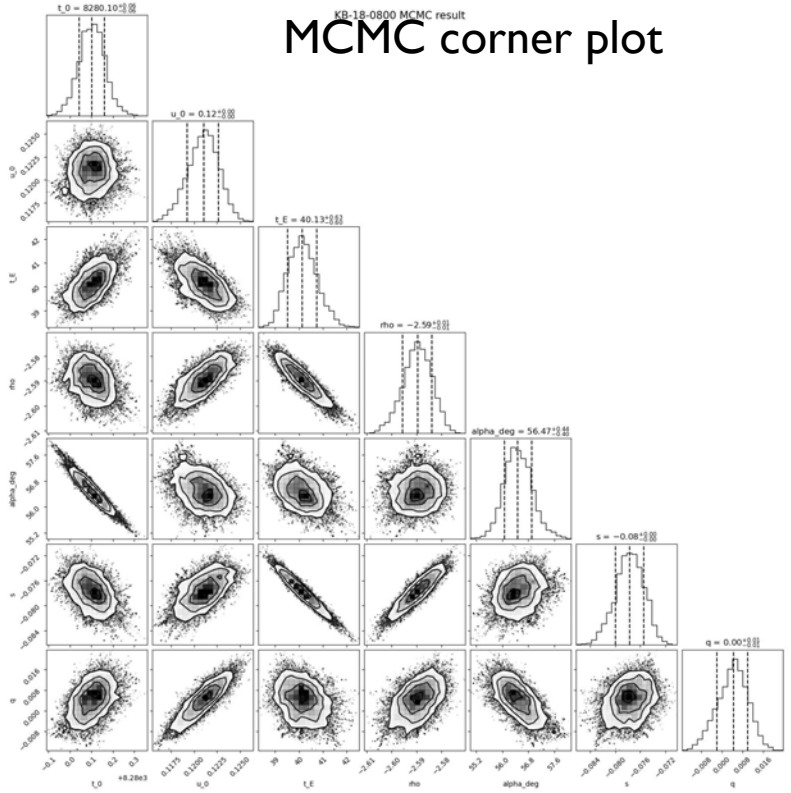
Credit:https://github.com/chi-feng/mcmc-demo

Hamiltonian Monte Carlo

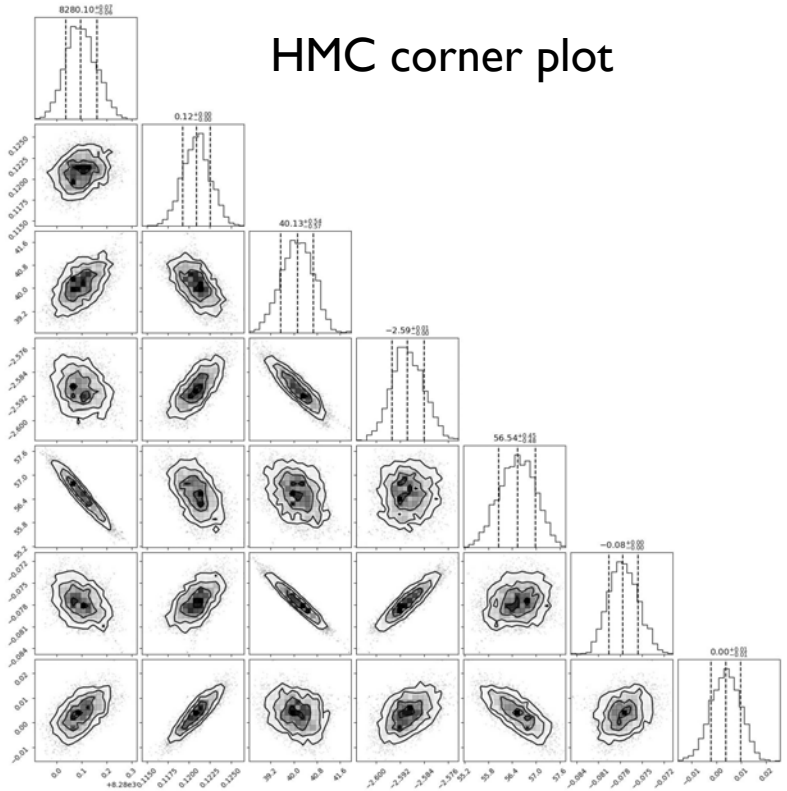
More Light curve evaluation : 248 (function + gradient) per ESS HMC 600 function per ESS MCMC.

ESS: Effective sample size

MCMC corner plot



HMC corner plot



Take Home Message

- We implement a fast, robust and accurate light curve calculation algorithm with Autograd in Jax

- With gradient we can explore more advanced algorithms:

  Basin hopping, Fisher matrix , HMC

- Future Plan

1. Optimization: Roots Finding etc.
2. High order effect : parallax etc.
3. Machine Learning
4. …

https://github.com/CoastEgo/BinaryJax