A fast approximation for the magnification of circular sources by point lenses

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Motivation

The Nancy Grace Roman Space Telescope, as well as other new infrastructures, is expected to lead to the discovery of thousands of free-floating planets (FFP) through microlensing. Therefore, efficient methods to fit the corresponding light curves will be required. In the present contribution, we present a very simple approximation for the Uniform Source Point Lens (USPL) case for arbitrary source sizes. We compare the accuracy and computational time of our method to the direct calculations of the Elliptical integrals involved [1] and also with the methods available from the public packages MulensModel and pyLIMA.

Exact solution and proposed approximation

The magnification of a uniform circular source is the ratio between the area of the images and the area of the source:

$$\mu = \frac{1}{\pi r^2}\int_0^{2\pi} y(\phi)\frac{dx(\phi)}{d\phi}d\phi \quad . \tag{1}$$

In the USPL model the resulting magnification can be expressed as [1].

$$\mu_{1,2} = \frac{1}{2\pi}\left[\pm\pi - K\left(\frac{\pi}{2}, m\right)\frac{(u-r)}{r^2}\frac{4+(u^2-r^2)/2}{\sqrt{4+(u-r)^2}} + E\left(\frac{\pi}{2}, m\right)\frac{u+r}{2r^2}\sqrt{4+(u-r)^2} + \Pi\left(n, \frac{\pi}{2}, m\right)\frac{2(u-r)^2}{r^2(u+r)}\frac{1+r^2}{\sqrt{4+(u-r)^2}}\right] \quad , \quad |u| \neq r \tag{2}$$

where

$$n = \frac{4ur}{(u+r)^2} \quad \text{e} \quad m = \sqrt{\frac{4n}{4+(u-r)^2}} \tag{3}$$

and

$$\mu = \frac{1}{\pi}\left\{\frac{2}{r} + \frac{1+r^2}{r^2}\left[\frac{\pi}{2} + \text{arcsen}\left(\frac{r^2-1}{r^2+1}\right)\right]\right\} \quad , \quad |u| = r \quad , \tag{4}$$

where u is the source position relative to the lens center in units of the Einstein radius and r is the size of the source in the same units.

The method applied in our approximation consists in using the power expansions given in [1] for high and small values of u/r, and an interpolation between the two for $u \sim r$:

$$\mu(u,r) = \begin{cases} \mu_+(u,r) \approx \sum_{m=0}^{4} c_m(u)\frac{r^m}{m!}, & \text{if } u > r + a \\ \mu_-(u,r) \approx \sum_{m=0}^{4} a_m(r)\frac{u^m}{m!}, & \text{if } u < a - r \\ \frac{\mu_+(u,r)+\mu_-(u,r)}{2}, & \text{if } |u-r| < a \end{cases} ,$$

where $a(r) = 0.04\,r$ is a choice that minimizes the difference with the exact result. Figure below show the **fractional difference** between the exact solution and the proposed approximation as a function of u and r:

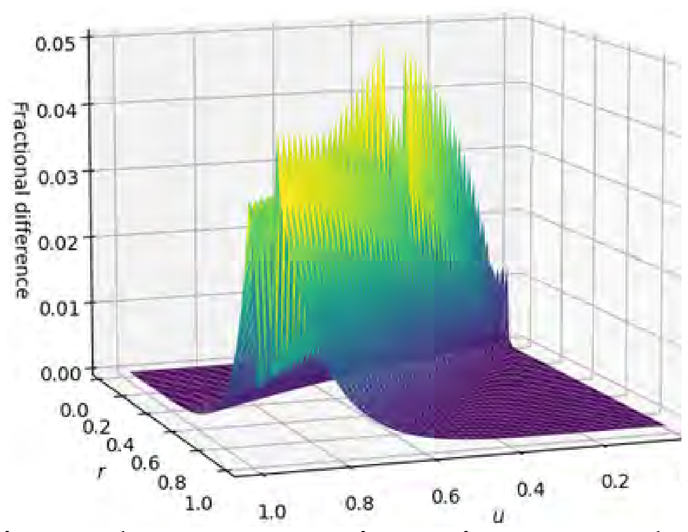


Fig. 1- Fractional difference between the approximation above and the exact result.

Comparing the approximation to the exact results, we obtain a **maximum** fractional difference of 5.22% in the range $0 < u < 1$ and $0 < r < 1$. The **mean** difference is 0.52% in this same range.

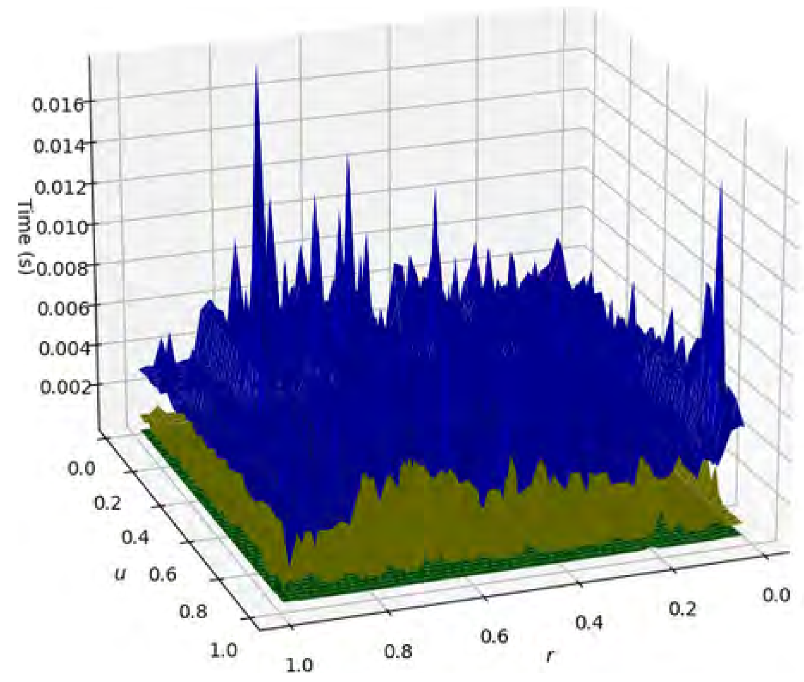
Time comparison

We compared the execution time by performing the direct integration of Eq. (1), using the analytic expressions Eq. (2)-(4) - using the standard numpy functions - and with the MulensModel [2] and PyLima [3] codes. The resulting time for computing 10^8 points in a grid in the range above, in a tabletop with core i3, are shown in the table below.

Temporal average

	Approximation	PyLima	MulensModel	Eq.'s (2)-(4)	Eq. (1)
$\overline{T}(s)$	7.5×10^{-5}	7.9×10^{-4}	4.4×10^{-3}	3.7×10^{-2}	7.4×10^{-2}
Relative Time	0.1%	1%	6%	50%	1

Fig. 2- time (s) x u x r for the 3 faster codes that we analysed: MulensModel (Blue), PyLima (Yellow) and our approximation (Green).



Light curve example

We use the 4 computations of the magnifications above to perform a χ^2 fit to the light curve of event KMT-2018-BLG-0244.

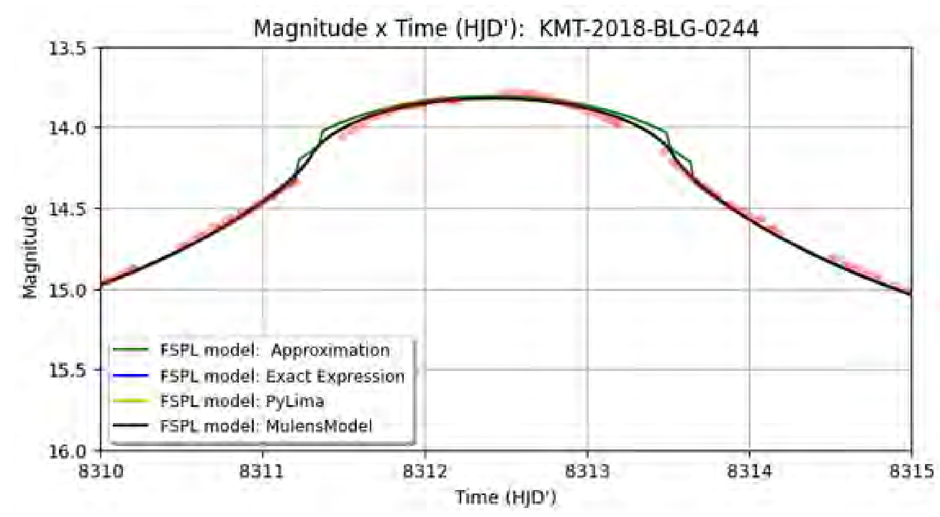


Fig. 3- Light curve fit to event KMT-2018-BLG-0244 using the 4 methods considered.

The results are shown in the table below.

	Approximation	PyLima	MulensModel	Eq.'s (2)-(4)
χ^2	45162	38976	38635	38658
χ^2/dof	19.1	16.5	16.3	16.3
r	0.237	0.235	0.236	0.236
u_0	0.141	0.149	0.149	0.149
t_0	8312.4	8312.4	8312.4	8312.4
t_E	6.0	6.1	6.1	6.1

The recovered parameters using the approximation to fit the light curve are in agreement within a few percent with the results from the exact computation.

Conclusion

- Our approximation for the USPL, based on the Mao & Witt (1994) expansions [1], is, on average, an order of magnitude faster than the fastest code we have tested (PyLima) and provides a good approximation in the whole range of source sizes and impact parameters.
- It can be used at least for an initial parameter search for fitting USPL light-curves, providing values close to those obtained with the exact expression.

References

[1]: Witt, H. J., & Mao, S. (1994). Can lensed stars be regarded as pointlike for microlensing by MACHOs?. The Astrophysical Journal, vol. 430, no. 2, pt. 1, p. 505-510, 430, 505-510.

[2]: Poleski, R., & Yee, J. C. (2019). Modeling microlensing events with MulensModel. Astronomy and computing, 26, 35-49. (https://github.com/rpoleski/MulensModel/)

[3]: Bachelet, E., Norbury, M., Bozza, V., & Street, R. (2017). pyLIMA: An Open-source Package for Microlensing Modeling. I. Presentation of the Software and Analysis of Single-lens Models. The Astronomical Journal, 154(5), 203. (https://github.com/ebachelet/pyLIMA)

[4]: KMTNet. KMT-2018-BLG-0244 Event. KMTNet.